1 SOME QUICK CALCULATION TECHNIQUES

1. Multiplication of an integer by 10 and powers of 10

To multiply an integer by 10 suffix it by a zero.
To multiply an integer by 100 suffix it by two zeros.
To multiply an integer by 10^n suffix it by n zeros.
123 x 10 = 1230, 123 x 100 = 12300 & 123 x 1000 = 123000 etc.

2. Multiplication of a decimal by 10 and powers of 10

To multiply a decimal by 10 shift the decimal by 1 place to right.
To multiply a decimal by 100 shift the decimal by 2 places right.
To multiply a decimal by 10^n shift the decimal by n places to right.
4.3261 x 10 = 43.261, 4.3261 x 100 = 432.61 & 4.3261 x 1000 = 4326.1 etc.

3. Division by 10 and powers of 10

To divide a number by 10 shift the decimal by 1 place to left.
To divide a number by 100 shift the decimal by 2 places of left.
To divide a number by 10^n shift the decimal by n places to left.
4123.4 $\div$ 10 = 412.34 , 4123.4 $\div$ 100 = 41.234 , 4123.4 $\div$ 1000 = 4.1234 etc.

4. Remainder of division by 9

To find the remainder when a number is divided by 9 we delete all 9's and then divide the sum of digits of the number by 9. The remainder so obtained in the answer.
Suppose we wish to find the remainder when 6936 is divided by 9.
We delete 9. Sum of remaining digits 6 + 3 + 6 = 15. When 15 is divided by 9 the remainder is 6. So remainder when 6936 is divided by 9 is also 6.

5. Multiplication by 5 and powers of 5

We know , $5=\frac{10}{2}, 25=\frac{100}{4}, 125=\frac{1000}{8}, 625=\frac{10000}{16}$ etc. $\left(5^n=\frac{10^n}{2^n}\right)$

To multiply a number by 5 suffix a 0 to the number and divide by 2.
To multiply the number by 25 suffix 2 zeros to the number and divide by 4.
To multiply a number by 5^n suffix n zeros to the number and divide by 2^n.
52 x 5 = 520 $\div$ 2 = 260 , 32 x 25 = 3200 $\div$ 4 = 800 ,
48 x 125 = 48000 $\div$ 8 = 6000 etc.

6. Division by 5 and powers of 5

To divide a number by 5 multiply it by 2 and divide it by 10 (shift decimal 1 place to left).
To divide a number by 25 multiply it by 4 and divide it by 100 (shift decimal 2 places to left).
To divide a number by 5^n multiply it by 2^n and divide it by 10^n (shift decimal n places to left).
52 $\div$ 5 = 52 x 2 $\div$ 10 = 104 $\div$ 10 = 10.4 ,
36 $\div$ 25 = 36 x 4 $\div$ 100 = 144 $\div$ 100 = 1.44
150 $\div$ 125 = 150 x 8 $\div$ 1000 = 1200 $\div$ 1000 = 1.2 etc.

7. Multiplication by 9, 99, 999 etc.

We know 9 = 10 - 1, 99 = 100 - 1, 999 = 1000 - 1 etc.

Let the number to be multiplied be 'A' . Then,
A x 9 = A x (10 - 1) = A x 10 - A
A x 99 = A x (100 - 1) = A x 100 - A
A x 999 = A x (1000 - 1) = A x1000 - A

23 x 9 = 23 x 10 - 23 = 230 - 23 = 207
43 x 99 = 43 x 100 - 43 = 4300 - 43 = 4257
28 x 999 = 28 x 1000 - 28 = 28000 - 28 = 27972

8. Finding Percentage

To find 1% of a number shift the decimal by two places to left.
To find 2% of a number multiply it by 2 and shift the decimal by two places to left.
To find a% of a number multiply it by a and shift the decimal by two places to left.
To find 5% of a number divide it by 20.
To find 10% of a number shift the decimal one place to left.
To find 20% of a number divide it by 5.
To find 25% of a number divide it by 4.
To find 50% of a number divide it by 2.
To find 12½% of a number divide it by 8.

To find $6\frac{1}{4}$% of a number divide it by 8.

To find 37½% of a number multiply it by $^{3}{}_{8}$.

To find $16\frac{2}{3}$% of a number divide it by 6.

To find $33\frac{1}{3}$% of a number divide it by 3.

To find $66\frac{2}{3}$% of a number multiply it by $\frac{2}{3}$.

9. Single line multiplication

2 digit number

		a	b
	x	c	d
Product			
Box number	3	2	1

Step 1 :- Find b x d. If it is a one digit number write it in box 1. If it is a two digit number write its unit digit in box 1 and carry over the tens place.
Step 2 :- Find a x d + b x c + carried over if any. If it is a one digit number write it in box 2. If it is a two digit number then write its units digit in box 2 and carry over the rest.
Step 3 :- Find a x c + carried over if any. Write the result in box 3.
Step 4 :- Read the number so obtained from left to right. This number is the required product.

		4	3
	x	2	7
Product	11	6	1
Box number	3	2	1

Step 1 :- 3 x 7 = 21, we write 1 in box 1 and carry over 2.
Step 2 :- 4 x 7 + 3 x 2 + 2 = 36. We write 6 in box 2 and carry over 3.
Step 3 :- 4 x 2 + 3 = 11. We write 11 in box 3.
Step 4 :- The required product is 1161.

Now, let us consider the multiplication of two three digit numbers

			a	b	c
		x	d	e	f
Product					
box number	5	4	3	2	1

Step 1 :- Find c x f. If it is a one digit number write it in box 1. If it is a two digit number write its units in box 1 and carry over the tens digits.
Step 2 :- Find b x f + c x e + carried over if any. If it is a one digit number write it in box 2. If it is a two digit number write its units digit in box 2 and carry over its tens digit.
Step 3 :- Find a x f + d x c + b x e + carried over if any. If it is a one digit number write it in box 3. If it is a two digit number write its units digit in box 3 and carry over its tens digit.
Step 4 :- Find a x e + b x d + carried over if any. If it is a one digit number write it in box 4. If it is a two digit number write its unit digit in box 4 and carry over the tens digit.
Step 5 :- Find a x d + carried over if any. Write the number so obtained in box 5.
Step 6 :- Now read the number from left to right. This number is the answer.

			2	3	5
		x	4	2	6
Product	10	0	1	1	0
box number	5	4	3	2	1

Step 1 :- 5 x 6 = 30. We write 0 in the box 1 and carry over 3.
Step 2 :- 3 x 6 + 2 x 5 + 3 = 31. We write 1 in box 2 and carry over 3.
Step 3 :- 2 x 6 + 4 x 5 + 3 x 2 + 3 = 41. We write 1 in box 3 and carry over 4.
Step 4 :- 2 x 2 + 4 x 3 + 4 = 20. We write 0 in box 4 and carry over 2.
Step 5 :- 4 x 2 + 2 = 10. We write 10 in box 5.
Step 6 :- The required product is 100110.

10.Calculating squares of numbers.

Squares of numbers ending in 5 can be found in the following manner.
Cross out 5 from the number to obtain the new number
Multiply this new number by its successor .(Next natural number.)
Suffix the product by 25 to obtain the square.
Let us find 75^2 using the above method. Deleting 5 from 75 we obtain 7. Successor of 7 is 8 . Multiplying 7 by 8 we get 56.Suffixing 56 by 25 we get the answer as 5625.

Squares of numbers from 51 to 59 can be found by the following pattern.
$\mathbf{51^2}$ = (25 + 1) x 100 + $\mathbf{1^2}$ = 2600 +1 = 2601.
$\mathbf{52^2}$ = (25 + 2) x 100 + $\mathbf{2^2}$ = 2700 + 4 = 2704.

Squares of numbers from 501 to 599 can be found by the following pattern.
513^2 = (250 + 13) x 1000 + **13^2** = 263169
534^2 = (250 + 34) x 1000 + **34^2** = 285156.

Square of number can be found the using identity $(a + b)^2 = a^2 + 2ab + b^2$.
This can be effectively used if the number formed by last two digits is small.(say between 1 and 10).
Let us find 503^2 using the above identity.
$503^2 = (500 + 3)^2 = 500^2 + 2 \times 500 \times 3 + 3^2$. = 250000 + 3000 + 9 = 253009.

Square of number can be found using the identity $(a - b)^2 = a^2 - 2ab + b^2$.
This can be effectively used if the number formed by last two digits is large.(say between 90 and 99).
Let us find 497^2 using the above identity.
$497^2 = (500 - 3)^2 = 500^2 - 2 \times 500 \times 3 + 3^2$. = 250000 - 3000 + 9 = 247009.

11. Methods of finding square root

Square root of perfect squares up to a four digit can be easily found by finding their units and tens digits. Let us first observe the following.

If units digit of a perfect square is 0 , units digit of its square root is 0.
If units digit of a perfect square is 1 , units digit of its square root is 1 or 9.
If units digit of a perfect square is 4 , units digit of its square root is 2 or 8.
If units digit of a perfect square is 5 , units digit of its square root is 5.
If units digit of a perfect square is 6 , units digit of its square root is 4 or 6.
If units digit of a perfect square is 9 , units digit of its square root is 3 or 7.

Step 1. Observe the units digit of the perfect square and determine the digit in the units place in the square root as discussed above. If the perfect square has 1 or 4 or 6 or 9 then there are two possible units digits.
Step2. Strike out from the right, last two digits of the number.. If nothing is left, we stop. The digit obtained in step 1 is the answer.
Step3. Now consider this left over number. We determine the largest one digit number whose square is less than or equal to this left over number. This is the tens digit of the square root.
If there are two possible answers then the correct answer can be found by actual multiplication.

Let us find the square root of 2209 by finding its units and tens digit:
Units digit is 9 so the units digit of the square root is either 3 or 7. (See the discussion above.).
let us strike out the two digits from the right. We get the number 22.
Square of 4 is the largest square that is less than 22 . ($4^2 = 16 < 22$ whereas $5^2 = 25 > 22$) Hence the tens digit of the square root is 4.
So square root of 2209 is either 43 or 47.
By actual multiplication $43^2 = 1849$ which is not equal to 2209. Hence square root of 2209 is 47.

12. Methods of finding cube of a number.

Let AB be a two digit number where A is the tens digit and B is the units digit. Let us form a row with 4 columns as follows

Box number	4	3	2	1

Step1. Find B^3. If it is a one digit number write it in box1. If it is a two or a three digit number write

its unit digit in box1 and carry over the rest .
Step 2. Find 3 x A x B ^{2} + carried over if any. If it is a one digit number write it in box 2. If it is a two or a three digit number write its unit digit in box 2 and carry over the rest.
Step3. Find 3 x A ^{2} x B + carried over if any. If it is a one digit number write it in box 3. If it is a two or a three digit number write its unit digit in box 3 and carry over the rest.
Step 4. Find A ^{3} + carried over if any .Write the number obtained in box 4 .
Step 5. We read the number from left to right. This is the required square.

Let us find the cube of 54 using column method.
Tens place is 5 and units place is 4 or A = 5 and B = 4 .

	157	4	6	4
Box number	4	3	2	1

Step1. $4^3 = 64$. We write 4 in box1 and carry over 6.
Step2. $3 \times 5 \times 4^2 + 6 = 246$. We write 6 in box 2 and carry over 24.
Step3. $3 \times 5^2 \times 4 + 24 = 324$. We write 4 in box 3 and carry over 32.
Step 4. $5^3 + 32 = 157$. We write 157 in box 4.
Step 5 The answer is 157464. Hence $54^3 = 157464$.

13. Methods of finding cube root.

Cube root of numbers up to a six digit perfect cubes can be easily found by finding their units and tens digits. Let us observe the following.
If a units digit of a perfect cube is 1, units digit of its cube root is 1 .
If a units digit of a perfect cube is 2, units digit of its cube root is 8 .
If a units digit of a perfect cube is 3, units digit of its cube root is 7 .
If a units digit of a perfect cube is 4, units digit of its cube root is 4 .
If a units digit of a perfect cube is 5, units digit of its cube root is 5 .
If a units digit of a perfect cube is 6, units digit of its cube root is 6 .
If a units digit of a perfect cube is 7, units digit of its cube root is 3 .
If a units digit of a perfect cube is 8, units digit of its cube root is 2 .
If a units digit of a perfect cube is 9, units digit of its cube root is 9 .
If a units digit of a perfect cube is 0, units digit of its cube root is 0 .

Step 1. Observe the units digit of the perfect cube and determine the digit in the units place in the cube root as discussed above.
Step2. Strike out from the right, last three digits of the number.. If nothing is left, we stop. The digit obtained in step 1 is the answer.
Step3. Now consider this left over number. We determine the largest one digit number whose cube is less than or equal to this left over number. This is the tens digit of the cube root.

Let us find the cube root of 300763 by finding its units and tens digit.
Units digit is 3 so the units digit of the cube root is 7. (See discussion above).
Let us strike out the three digits from the right. We get the number 300.
Cube of 6 or 216 is the largest cube that is lass than 300 . ($6^3 = 216 < 300$ whereas $7_3 = 343 > 300$) Hence the tens digit of the cube root is 6 .
So cube root of 300763 is 67.

14. Mathematical conversions

Conversion of Length

mm → cm	$\div 10$	cm → m	$\times 10$
cm → dm	$\div 10$	dm → cm	$\times 10$
dm → m	$\div 10$	m → dm	$\times 10$
m → km	$\div 1000$	km → m	$\times 1000$
cm → m	$\div 100$	m → cm	$\times 100$

Conversion of Area

$mm^2 \rightarrow cm^2$	$\div 100$	$cm^2 \rightarrow mm^2$	$\times 100$
$cm^2 \rightarrow dm^2$	$\div 100$	$dm^2 \rightarrow cm^2$	$\times 100$
$dm^2 \rightarrow m^2$	$\div 100$	$m^2 \rightarrow dm^2$	$\times 100$
$m^2 \rightarrow$ ha	$\div 10000$	ha $\rightarrow m^2$	$\times 10000$
ha $\rightarrow km^2$	$\div 100$	$km^2 \rightarrow$ ha	$\times 100$
$cm^2 \rightarrow m^2$	$\div 10000$	$m^2 \rightarrow cm^2$	$\times 10000$
$m^2 \rightarrow km^2$	$\div 1000000$	$km^2 \rightarrow m^2$	$\times 1000000$

Conversion of Speed

m/s → km/h	$\times \frac{18}{5}$ or 3.6	km/h → m/s	$\times \frac{5}{18}$

Conversion of Volume

(Note : cm^2 = m*l*, dm^3 = *l* , m^3 = k*l*)

m*l* → *l*	$\div 1000$	*l* → *ml*	$\times 1000$
l → k*l*	$\div 1000$	*k* → *l*	$\times 1000$
ml → k*l*	$\div 1000000$	*k* → *ml*	$\times 1000000$

Conversion of Time

sec → min	$\div 60$	min → sec	$\times 60$
min → hrs	$\div 60$	hrs → min	$\times 60$
sec → hrs	$\div 3600$	hrs → sec	$\times 3600$

Conversion of mass

gm → kg	$\div 1000$	kg → g	$\times 1000$
kg → quintal	$\div 100$	quintal → kg	$\times 100$
ton → quintal	$\div 10$	quintal → ton	$\times 10$

Q1. Without actual multiplication find the value of

a) 43 x 10 b) 235 x 100

c) 143 x 1000 d)745 x 10000

e) 6432 x 100000

Q2. Without actual multiplication find the value of

a) 2.357 x 10 b) 12.625 x 100

c) 13.7291 x 1000 d) 143.62957 x 10000

e) 0.000432 x 100000

Q3. Without actual division find the value of

a) $24.8 \div 10$ b) $1143.2 \div 100$

c) $2356.7 \div 1000$ d) $119.26 \div 10000$

e) $432.15 \div 100000$

Q4. Without actual multiplication find the value of

a) 23 x 5 b) 432 x 25

c) 128 x 125 d) 1616 x 625

e) 3264 x 3125

Q5.Without actual division find the value of

a) $23 \div 5$ b) $108 \div 25$

c) $231 \div 125$ d) $436 \div 625$

e) $7129 \div 3125$

Q6. Without actual multiplication find the value of

a) 23 x 9 b) 24 x 99

c) 25 x 999 d) 26 x 9999

e) 27 x 99999

Q7. Without actual division find the remainder when the following numbers are divided by 9

a) 42365 b) 112233541

c) 231231843 d) 342342342

e) 1051057

Q8. Without actual calculation find the required percentage in the following

a) 1% of 235 b) 2% of 270

c) 6% of 435 d) 5% of 75

e) 10% of 450 f) 20% of 2360

g) 25% of 4440 h) 50% of 734

i) 12 ½ % of 96 j) 37 ½ % of 48

Q9.Multiply the following using single line multiplication

a) 23 x 37 b) 49 x 28

c) 43 x 615 d) 712 x 53

e) 113 x 215

Q10. Without actual multiplication find the squares of the following.

a) 85 b) 95

c) 995 d) 105

e) 1005

Q11. Without actual multiplication find the squares of the following.

a) 53 b) 57

c) 517 d) 527

e) 299 f) 399

g) 401 h) 901

Q12.Use column method to find the squares of the following numbers.

a) 78 b) 43

c) 67 d) 59

e) 89

Q13. Find the squares of the following using diagonal method

a) 79 b) 53

c) 112 d) 231

e) 752

Q14.Find the square root of the following by finding their units and tens digits.

a) 1444 b) 2209

c) 4761 d) 8836

e) 5929

Q15. Find the cube of the following using column method

a) 23 b) 31

c) 37 d) 41

e) 43

Q16. Find the cube root of the following by finding their units and tens digits.

a) 42875 b) 103823

c) 13824 d) 117649

e) 140608

Q17. Do the following conversions

a) 40mm = ….cm b) 30 cm = ….dm

c) 70 dm = ….m d) 4000 m = ….km

e) 500 cm = ….m f) 6 cm = ….mm

g) 17 dm = ….cm h) 4 m = ….dm

i) 2 km = ….m j) 7 m = ….cm

Q18. Do the following conversions

a) 400 mm^2 = ….cm^2 b) 700 cm^2 = ….dm^2

c) 4000 cm^2 = ….m^2 d) 40000 m^2 = ….ha

e) 5000000 m^2 =...km^2 f) 40000 ha = ….km^2

g) 5 ha = ….m^2 h) 5 m^2 = ….cm^2

Q19. Do the following conversions

a) 5000 ml = ….l b) 3000 l = ….m^3

c) 4 m^3 = ….l d) 20 l = ….ml

e) 90 m/ s = ...km / h f) 54 km/ h = ….m/s

Q20. Do the following conversions

a) 4000 gm = ….kg b) 200 kg = ….q

c) 70 q = ….tons d) 5000 kg = ….tons

e) 7 tons = ….q f) 50q = …kg

g) 2 tons = ….kg h) 5 kg = ….gm

Answers

Q1a	430	I	12	c	24
b	23500	j	18	d	49
c	143000			e	52
d	7450000	Q9 a	851		
e	643200000	b	1372	Q17 a	4
		c	26445	b	3
Q2 a	23.57	d	37736	c	7
b	1262.5	e	24295	d	4
c	13729.1			e	5
d	1436259.7	Q10 a	7225	f	60
e	43.2	b	9025	g	170
		c	990025	h	40
Q3 a	2.48	d	11025	I	2000
b	11.432	e	1010025	j	700
c	2.3567				
d	0.011926	Q11 a	2809	Q18 a	4
e	0.0043215	b	3249	b	7
		c	267289	c	4
Q4 a	115	d	277729	d	4
b	10800	e	89401	e	5
c	16000	f	159201	f	4
d	1010000	g	160801	g	50000

Q5 a	4.6	Q12 a	6084	Q19 a	5
b	4.32	b	1849	b	3
c	1.848	c	4489	c	4000
d	0.6976	d	3481	d	20000
e	2.28128	e	7921	e	324
				f	15
Q6 a	207	Q13 a	6241		
b	2376	b	2809	Q20.a	4
c	24975	c	12544	b	2
d	259974	d	53361	c	7
e	2699973	e	565504	d	5
				e	70
Q7 a	2	Q14 a	38	f	5000
b	3	b	47	g	2000
c	0	c	69	h	5000
d	0	d	94		
e	1	e	77		
Q8 a	2.35	Q15 a	12167		
b	5.4	b	29791		
c	26.1	c	50653		
d	3.75	d	68921		
e	45	e	79507		
f	472				